

### Title of the research project:

Singular stochastic control models with applications: from theory to applications and back

### Keywords (up to five)

Stochastic control, stochastic differential equations, partial differential equations, power-systems management, energy markets

### Supervisors (at least two from two different areas):

*Supervisor 1 (name, contact details, homepage, area of expertise)*

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Stochastic control, mathematical finance

### Project description (max 5000 characters)

*Please include a description of the work to be carried out. State of the art, key research questions and project objectives, workplan and the methodological and application aspects of the project.*  
[add a page if needed]

#### *Key research questions:*

This project concerns the study of stochastic systems that are singularly controlled and share two main features:

1. The controlled dynamics may incur absorption in some parts of the state space;
2. The marginal yield obtained from optimally exerting control fails to be monotone.

Models of this kind arise naturally in a number of fields where an agent is able to control a stochastic dynamics for as long as the corresponding state process evolves inside a certain admissible subset of the state-space. The lack of monotonic marginal yields arises when the cost of exerting control is “state-dependent”, that is, it varies with the current state of the controlled system. The combination of these two features leads to the study of complex optimization problems that have not yet been addressed in the mathematical literature.

#### *Applications:*

There is a vast range of applications for these models:

- (i) Economic-financial valuation of renewable energy storage facilities: the owner of the storage facility chooses the charging/discharging policies taking into account the spot price of electricity, high unpredictability of energy markets, environmental policies, and climate changes. These features lead to state-dependent cost of control.

- (ii) Budget/capacity constraints: in the same framework as in (i) (and more in general in economic models) adding a budget constraint leads to considering the absorption feature for the underlying stochastic dynamics. For example, absorption occurs at the time of bankruptcy of the economic agent or it can be used to model finite storage capacity.
- (iii) Resource extraction/optimal harvesting models: these are models for the optimal management of depletable resources. The cost of control is state-dependent because it reacts to the scarcity of the resource being harvested/mined. Absorption corresponds to the exhaustion of a given resource or (in more virtuous models) to sustainability caps imposed by environmental regulators
- (iv) Power-systems management: in networks operating with a mix of conventional generation and renewables we can use singular controls to describe the use of conventional generators. That is needed to meet the demand in excess of generation from renewable sources during peak times. The cost of control is state dependent because there are physical constraints on the amount of conventional generation that can be made available to the network operator depending on the notice period. Absorption may be used to describe partial/total black-outs due to failing to meet the excess demand.

*The main objectives are:*

1. the design of stochastic control models suitable for real-world applications described above;
2. the theoretical analysis of the resulting mathematical problems;
3. numerical implementation of optimal strategies;
4. calibration to real-data and back-testing

*Methodology:*

At the mathematical level these problems can be addressed both theoretically and numerically. At the theoretical level one can employ a blend a probabilistic and analytical methods drawing on the theory of stochastic calculus and on partial differential equations and free boundary problems. At the numerical level one can use simulation with Least-Squares Monte Carlo methods and approximation algorithms for partial differential equations. Recent developments concerning the use of machine learning and deep learning for the solution of high-dimensional partial differential equations may be explored.

*Workplan:*

Initially the student will explore existing challenges in the areas described in “Applications”. Once she/he has picked the potential application of interest and a class of suitable open problems is identified the student will proceed to a rigorous mathematical formulation of the stochastic control model. A theoretical study of the latter will follow, in order to determine conditions under which the problem is mathematically well-posed, in the sense that optimal controls exist with finite total value of the optimization criterion. Once that is accomplished, it will be possible to proceed along two directions: either determining theoretical constructive procedures to obtain optimal controls in closed form or designing suitable numerical algorithms for the numerical evaluation of the optimal controls. Where possible, the outputs of the theoretical models will be tested against real data after suitable calibration.

### **Relevance to the MERC PhD Program (max 2000 characters)**

*Briefly describe how this project fits within the scope of the MERC PhD program describing its interdisciplinary aspects, relevance in application and beneficiaries.*

The project addresses specifically questions arising in engineering-related fields with an approach and methodology that belong to the mathematical area. So, it is multi-disciplinary in nature and motivated by applications in the real world.

Models use stochastic dynamics in continuous time. This captures the “risk and complexity” part of the PhD program via randomness and the impossibility to control/predict the trajectory of the system in an exact manner.

Potential beneficiaries of these models include electricity network operators, market regulators and policymakers. Indeed, an accurate evaluation of expected operational costs and optimized control strategies are likely to strengthen existing protocols in all areas of application identified above.

### Key references

Chiarolla, M.B., Ferrari, G., Stabile, G., 2015. Optimal dynamic procurement policies for a storable commodity with Levy prices and convex holding costs, *European Journal of Operational Research*, 247, pp. 847-858.

Chiarolla, M.B., Haussmann, U.G., 2009. On a Stochastic, Irreversible Investment Problem, *SIAM Journal on Control and Optimization*, 48, pp. 438-462.

De Angelis, T., and Ekstrom E., 2017. The dividend problem with a finite horizon. *Annals of Applied Probability* 27 (6), pp. 3525-3546

De Angelis, T., 2020. Optimal dividends with partial information and stopping of a degenerate reflecting diffusion. *Finance and Stochastics* 24(1), 71-123

Karatzas, I. and Shreve, S.E., 1998. *Methods of mathematical finance* (Vol. 39). New York: Springer.

Karatzas, I. and Shreve, S.E., 1998. *Brownian Motion and Stochastic Calculus*. Springer, New York, NY.

Yong, J. and Zhou, X.Y., 1999. *Stochastic controls: Hamiltonian systems and HJB equations* (Vol. 43). Springer Science & Business Media.

### Joint supervision arrangements

*Describe joint supervision arrangements, e.g. weekly/monthly meetings with one or both supervisors, how will the joint supervision be split etc.*

Weekly supervision meetings with at least one supervisor (to be held online). In person meetings will be scheduled on average every 6 weeks for a closer monitoring of student’s progress.

### Location and length of the study period abroad (min 12 months)

Give details of the foreign research institution where the student will be host together with the full name and contacts of the foreign host. Please indicate if the foreign institution has already agreed to host the student and when the student is expected to travel abroad.

This is not yet arranged and it will depend on the direction of work undertaken by the student (theoretical/numerical/applied). Possible host institutions include:

1. University of Uppsala (Sweden). Host: Prof. E. Ekstrom
2. Toulouse School of Economics (France). Host: Prof. S. Villeneuve
3. University of Paris Dauphine (France). Host: Prof. R. Aid
4. University of Leeds (UK). Host: Dr J. Palczewski (Associate Prof.)
5. University of Warwick (UK). Host: Dr G. Liang (Associate Prof.)

It is expected that the student will travel abroad in the second half of the second year from the start of the supervision. This should guarantee that the research plan has been laid out clearly and the main methodological tools are at an advanced stage of their development.

All potential host institutions and host academics are collaborators of the proposed supervisors and similar exchanges of students have already taken place in the past. So, we anticipate no difficulty concerning that.

#### Any other useful information

E.g. involvement of stakeholders, industrial partners, other research institutions etc, funded research projects related to the proposed activity etc

It may be possible to start a collaboration with C. Alasseur at FiME lab in Paris (Laboratoire de Finance des Marchés de l'Energie) and develop links to Électricité de France (EDF). EDF is project partner on an EPSRC Early Career Fellowship, submitted by T. De Angelis and currently under review with EPSRC in the UK.

This is an ambitious plan which carries some uncertainty and the exact development of the PhD project depends strongly on the attitude and scientific interests of the student.

***Please return this form via email by no later than 3rd May 2021 to merc@unina.it***