SSM
Scuola Superiore Meridionale

Course title:

Numerical treatment of PDEs

Duration [number of hours]**: 24**

PhD Program [MERC/MPS/SPACE]**: MPHS**

Name and Contact details of unit organizer(s):

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Course Description [max 150 words]**:**

Aim of this course is to introduce the basic theory for the numerical approximation of partial differential equations. A review on existing methods is given, then focus is on the treatment of elliptic linear problems with the Finite Element Methods.
Also, some insights on the treatment of time derivatives for parabolic and hyperbolic problems is given. Matlab and FreeFem coding are introduced and used during all the course.

Syllabus [itemized list of course topics]**:**

- Introduction on PDEs and numerical approaches for the discretization. Abstract formulation. Hilbert spaces, Riesz representation theorem, Lax-Milgram
- Essential notions on Sobolev spaces. Variational formulation, Ritz-Galerkin method, Cea lemma. Weak formulation of elliptic problems: derivation of models, treatment of both essential and natural boundary conditions.
- Galerkin-Finite Elements Methods. Conformal methods, meshing, the choice of the finite element. The Lagrangian Elements on triangularizations.
- Interpolation error: definition of the interpolator; Deny-Lions theorem; related finite elements and reference element. Global estimate. Best approximation properties of Galerkin methods in the symmetric case: strain energy, potential energy, numerical stiffness, discrete eigenvalues.
- Error estimate in the Poisson case both in norm H^1 and L^2 (Aubin-Nitsche).
- The structure of a finite element code. Meshing and change of variables in the reference domain. Local construction and global assembly.
- Matrix description of Finite Element Method,
- From finite elements to polytopal methods. Polytopal methods support much more general meshes than standard finite elements, and their construction is fully discrete, thus removing the need for an underlying space of functions. We will, in particular, focus on the Hybrid High-Order (HHO) method applied to a scalar diffusion problem, for which we will provide a derivation and a complete stability and convergence analysis.

Assessment [form of assessment, e.g., final written/oral exam, solutions of problems during the course, final project to be handed-in, etc.]**:**

Projects will be proposed during the course, along with some insights from books or recent literature.

Suggested reading and online resources:

Suggested books:

1. S. Brenner & L. Scott “The Mathematical Theory of Finite Element Methods“, Springer 2008
2. A. Quarteroni ”Numerical models for differential problems “, Springer 2016
3. T. Hughes “The finite Element Method”, Dover 1987 Book 2
4. D. A. Di Pietro and J. Droniou, The Hybrid High-Order method for polytopal meshes, Number 19 in Modeling, Simulation and Application, Springer International Publishing, 2020. DOI: 10.1007/978-3-030-37203-3

Notes: https://www.mate.polimi.it/biblioteca/add/qmox/49-2013.pdf

Sites: https://freefem.org/